Stability and time duration record of an OAM multiplexing scheme in a highly disturbed environment.

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We report the results of a series of real-world experiments showing a long stability in time of wide-band digital multiplexing (MUX) schemes in a point-to-point (p2p) configuration based on electromagnetic waves carrying Orbital Angular Momentum (OAM) in the presence of external disturbing signals. Two different and independent links, both with channel capacity doubled by the presence of an $\ell = 1$ OAM beam, were realized indoor and worked for 5 months uninterruptedly. The links were realized during the exhibition Globale–Digitale [1] in a large closed environment, crowded with people, where were present disturbing radio signals from the local wifi, bluetooth systems and additional interferences caused by the reflections from walls and metal structures. In all the links two wide–band analog TV channels, both with horizontal polarization in the same frequency band and FM-carrier centered at 2.414 GHz with 27 MHz bandwidth were transmitting high-definition audio-video channels. The quality of the TV transmissions remained stable during the whole time slots. Periodical tests to measure the separation between the two channels gave averaged value of 33 dB and 25 dB for the first and second link. We characterized the channel capacity and stabiity with additional tests using a 4-QAM digital transmission in the twisted channel, where we obtained a maximum channel separation value of ~ 50 dB when the conditions of phase orthogonality were fulfilled and no reflections were present. Our results show the practical feasibility of setting up for long periods in time and at different distances stable OAM radio/TV links in the real world where reflections, interferences from disturbing signals in the same frequency band are present, paving the way for for secure and efficient communication schemes under electromagnetic jamming conditions.

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INTRODUCTION

Current radio science and communication implementations based on the electromagnetic (EM) linear momentum (Poynting vector) physics layer are beginning to approach their limits in terms of radio frequency spectrum availability and occupancy. This calls for the introduction of new radio paradigms. For this purpose, it has been proposed that the EM angular momentum, well described in the standard literature [3–17, 19, 22, 23] but hitherto underutilized in radio science and technology, be fully exploited in radio communications to help and improve the existing multiplexing systems [2, 24–28].

Electromagnetic (EM) radiation carry not only energy and linear momentum, associated with translational dynamics and force action of the system, but also carry angular momentum, associated with rotational dynamics and torque action of the same system. Present–day radio science and communication implementations based on the EM linear momentum Poynting vector) physics layer are beginning to approach their limits in terms of radio frequency spectrum availability and occupancy even with the use of multiple-input multiple-output (MIMO) techniques [18]. The use of spin angular momentum (SAM) in the form of wave polarization can at most double the capacity of a linear momentum-based communication within a given oscillation frequency bandwidth. This calls for the introduction of new radio paradigms. For this purpose, it has been proposed that the EM angular momentum, well described in the standard literature [19, 23] be fully exploited in radio communications either for stable multiple-input-multiple-out (MIMO) systems with linear momentum radio technologies or with new types of antennas capable to receive OAM [29].

Proof-of-concept studies have shown that it is possible to use the total angular momentum, *i.e.* SAM+OAM, as a new physical layer for radio science and technology exploitation, not only the linear momentum aspect of OAM beams as performed in MIMO-like systems [20, 21] that are the core of most of the OAM experiments nowadays. These studies include numerical experiments [24, 26] showing that it is feasible to utilize OAM in radio; controlled anechoic chamber laboratory experiments [27] verifying that it is possible to generate and transmit radio beams carrying non-integer OAM and to measure their OAM spectra in the form of weighted superpositions of different integer OAM eigenstates; and outdoor experiments started with the pioneering works in Ref. [2, 28, 30] verifying that in a real-world setting different signals, encoded in different OAM states, can be transmitted independently to a receiver located in the (linear momentum) far zone and be resolved there. Differently from MIMO systems, where a stable channel separation is achieved through static phase differences between different paths, OAM can support dynamical reading schemes based on the OAM rotational doppler effect [31, 32]. As has been demonstrated in experiments at optical frequencies, the use of EM angular momentum can indeed increase the capacity of wireless communications [35–39] and in the vast literature that can be found nowadays.

As follows from Maxwell's equations, OAM is a pseudovector, associated with a variation of the phase of the EM field as a function of the azimuthal angle, φ, around the beam axis, discretized such that an EM field carrying an arbitrary amount of angular momentum is a superposition of a denumerably infinite set of discrete OAM modes, such as Laguerre-Gaussian (LG) modes, each of which is proportional to $\exp(\mathrm{i}m\varphi), m = 0, \pm 1, \pm 2, \ldots$ and identified by the integer m, called topological charge. [19, 40, 41]. These OAM modes are mutually orthogonal and are therefore independent. If different OAM beams are independently generated in the same frequency band, they do not interfere with each other [42] and can therefore be used to encode, at least in principle, a much larger amount of information (ideally infinitely many) onto any part of an EM beam, down to the individual photon level [43]. OAM manifests itself as different EM field phases in one and the same direction at different points in the beam front. Moreover, reflections change the sign of each OAM state and a reflected beam is orthogonal with respect to the initial one, becoming a signal with different OAM value.

In this paper, we report the results of the tests for the stability in time of a multiplexing system based on OAM waves for wide-band radio/TV transmission indoor and study the stability of the MUX system through the possible degradation in time of the signals transmitted in the presence of multiple reflections and disturbing signals. This situation represents a common situation that can be found in a multiplexed TV link or in an indoor video channel transmission.

EXPERIMENTAL RESULTS

Here we report the experimental results of the two channel multiplexing scheme based on OAM waves operating uninterruptedly for five months during the Globale-Digitale international exhibition. We also report the tests for the transmission capacity of the OAM channel and the channel insulation capacity in presence of an external disturbing signal measured with the use of a QPSK DVB-S digital signal. The dual link was also used as a part of an artistic installation to transmit two high definition TV channels to visualize, in two separate screens, high quality movies with artistic and scientific contents for the exhibition, started in September 2015 (for more information see Ref. [44]). During the long run more than 20.000 people interacted with the video installation. The same link was then set up again for additional tests in campaigns of three months in 2018 and 2019 that confirmed the stability of the link with respect to external disturbances.

Analog wide–band dual OAM TV channel transmission indoor

Most of the OAM modes are physical states of the EM field that preserve their topological properties during their propagation in free space. They represent a viable solution for a stable MUX realized with standard linear momentum–based techniques as the waveform always propagate with the same stable topology [45]. This is confirmed by the results of a recent work where the topological stability of radio vortices at 233GHz permitted to measure the rotation of the supermassive black hole in the galaxy M87, after having traveled a distance of 56 millions of light years in the outer space [46].

To test the stability in time of an OAM-based MUX transmission and reception scheme we decided to adopt a 27 MHz wide-band analog channel scheme transmitting two high quality audio/video TV channels at the frequency of 2.414GHz, with horizontal linear polarization, in an indoor environment in the presence of reflections. The configuration is point-to-point (p2p) with maximum output power of 0.1W.

The OAM link was realized with an $m = 1$ twisted beam and the other untwisted. We generated the twisted beam with the helicoidally deformed parabolic antenna used for the 442 m link in the Venice experiment [2], whereas the untwisted beam was generated with dipole in a $\lambda/4$ retroreflector backfire configuration. Because of the unique and different geometric spatial shapes of the respective phase fronts of the two beams, the field vectors of the linearly polarized $m = 1$ twisted beam will be in anti-phase at two points on diametrically opposite sides of the OAM phase singularity at the centre of the two aligned beams and orthogonal to the propagation axis, whereas they will be in phase for an $m = 0$ beam in the same reception plane [2, 24, 27]).

The MUX of two channels transmitted for 4 months without any interruption and without showing signal degradation to all the visitors of the 2015 Globale-Digitale exhibition in Karlsruhe [1] and 3 additional months in 2018 and 2019. Following Ref. [2] as shown in Fig. 1, in the indoor link, we used two collinear, linearly polarized radio beams, one twisted ($m = 1$)and one untwisted ($m = 0$). Horizontal linear polarization was used. The OAM $m = 1$ state was produced by the helicoidally deformed parabolic antenna used in the experiment reported in Ref. [2]. The untwisted linear-momentum ($m = 0$) signal at the same carrier frequency and with the same polarization was emitted by a dipole in backfire configuration with retroreflector at $\lambda/4$ mounted on the top of the parabola. The two antennas were mounted on a vertical configuration to minimize the advantages of a standard MIMO scheme to discriminate one channel with respect to the other. Both channels were fed by an FM signal each from two transmitters operating at a carrier frequency of 2.414 GHz with emitting power of 10mW.

At the receiving end, at a distance of $\sim 200\lambda$ from the transmitting antennas, the instantaneous EM fields of the $m = 0$ and $m = 1$ modes result in phase on one side of the central axis (azimuthal angle $\varphi = 0$) and in anti-phase at the opposite side ($\varphi = \pi$). Therefore two simple standard linear-momentum interferometers, in anti-phase with respect to each others were used to discern between the twisted and untwisted beams.

The interferometers shared the same baseline and the receiving antennas were separated horizontally by a quantity $\sim 20\lambda$ and, vertically, in each arm, the antennas of interferometer 1 were separated by 2.5λ from the respective antennas of interferometer 2.

To minimize the possible cross-talking between the antennas of the two interferometers in each arm and antenna resonances, we used two couples of different types of antennas with the

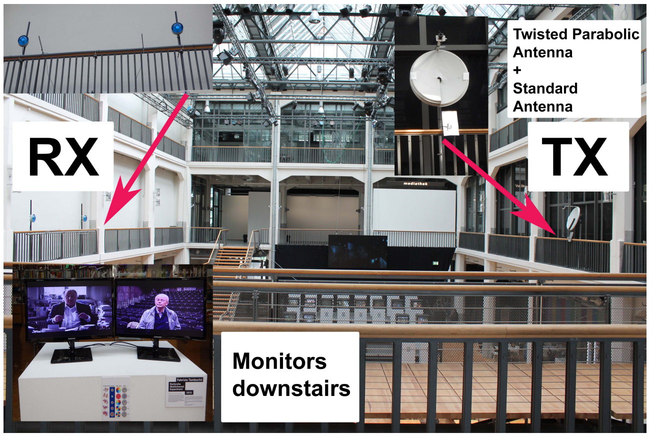

FIG. 1. The indoor experimental setup in the foyer of ZKM. The MUX of two FM wide-band channels transmitted for 4 months without any interruption did not show signal degradation. Horizontal linear polarization was used. The OAM $m = 1$ state was produced by the helicoidally–deformed parabolic antenna used in the experiment reported in Ref. [2]. The untwisted linear–momentum ($m = 0$) signal at the same carrier frequency and with the same polarization was emitted by a dipole in backfire configuration with retroreflector at $\lambda/4$ mounted on the top of the parabola. Two interferometers, sharing the positions of the respective two arms, in anti-phase with respect to the other, were used to discern between the twisted and untwisted received beams. The first was made with a couple of backfire antennas and the other with a couple Yagi-Uda antennas.

same gain. The first interferometer was realized with two identical 26 cm (2.1λ) diameter 16 dBi backfire antennas, connected together trough a signal splitter/combiner. The second interferometer was instead realized with a couple of 16 dBi Yagi-Uda antennas, also connected together trough another signal splitter/combiner.

In both interferometers a phase tuner, in the form of a silver slit waveguide coupled to a signal circulator, was inserted into one of the interferometer arms. By moving the cursor in the slit waveguide, we retarded the signal received by one of the two interferometer antennas relative to the other. Once regulated the positions of the cursor in the waveguides to optimize the phase tuning, they remained fixed during the whole transmission. No electronic devices were used to adjust and tune the phase in case of error during the run.

The requirement for our wide-band transmission was a channel separation with carrier-to-noise ratios of at least ~ 20 dB. Down to that value, the presence of continuous interferences would affect the transmission with very evident effects such as the rise of patterning on the video, with lines and/or "snow" arriving to the complete blocking of the signal with a frozen image. Instead, the presence of a Wi-Fi, would cause horizontal noise bars on the screen, with and noise in the audio channel. No macroscopic effect due to interferences have been reported during the 4 months long run.

The experimental results reported here show that a multiple-channel TV transmission using OAM states is compatible and robust with respect to reflections, interference from a beam with different OAM value and does not need error correction code protocols like those used in digital channels. More interestingly, the advantages offered by the use of digital multiplexing techniques, even those based on phase-coding such as phase shift keying (PSK) [30, 33, 34], are preserved and can be used to further improve the channel capacity also in these complicated environmental situations, as shown in the tests reported below, arriving up to 1024QAM at 8GHz at the distance of a hundred of meters.

Fading caused by reflections is one of the most common complications in wireless communications. Deflections from walls, ground and other objects can have a detrimental effect on a communication channel and decrease the quality of the transmission. On the other hand, reflections can also be utilized in multi-path linear-momentum communication protocols such as MIMO (Multiple-Input-Multiple-Output) to increase the signal to noise radio and to enhance the information transfer capability; see the discussion in ref. [**?**].

Unlike the standard multiport techniques that exploit EM linear momentum, the problem will be more intricate and subtle using OAM states, because each reflection will introduce a parity change and OAM channel swapping from left- to right-handed twist and *vice versa*. For the purpose of assessing the robustness of the information transfer against perturbations, we used the reflection of the waves off the ground with horizontal polarization, since this maximizes the fading.

OAM TV channel capacity test with digital transmissions.

We now test the radio link and measure the bit transfer rate with a digital channel in situations similar to those found transmitting TV OAM-channels in an urban setting. TV channels usually experience reflections and interference with other disturbing signals that may overlap the same frequency band.

EM beams that are physically encoded with angular momentum have a particular, azimuthally dependent spatial phase distribution and the OAM signature changes in presence of reflections: OAM is a pseudovector. OAM transmission is compatible and robust with respect to digital multiplexing techniques based on phase coding such as phase shift keying (PSK). This is true also when the OAM signal is disturbed by the presence of a strong wide-band interfering signal on the same carrier frequency and by the presence of reflections. Following [30], we investigate what impact these physical effects might have on the use OAM in TV channels characterizing the channel capacity and insulation by using known digital phase modulation protocols also when reflections and disturbing signals are present.

In the experiment described here, we used two collinear, linearly polarized radio beams, one twisted ($m = 1$)and one untwisted ($m = 0$). As shown in Fig. 3, we generated the twisted beam with a helicoidally deformed parabolic antenna, whereas the untwisted beam was generated with a Yagi-Uda antenna, designed for the UHF S-band carrier frequency used. Because of the unique and different signatures of the respective phase

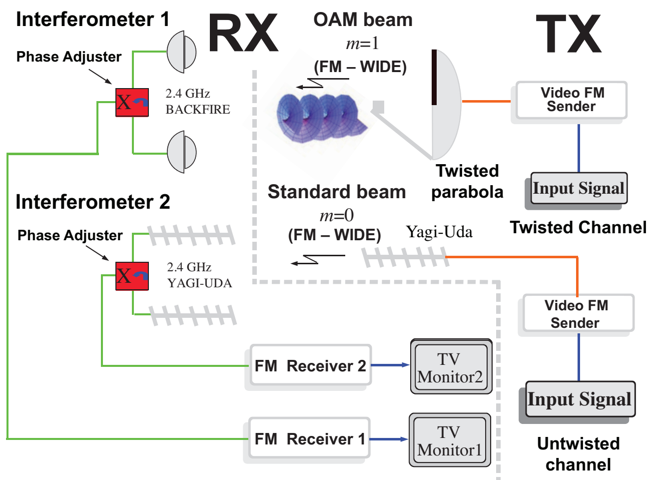

FIG. 2. Schematic description of the experimental setup. The two high definition video channels were transmitted simultaneously in the same 2.414 GHz frequency band with the same horizontal polarization state. The channel was transmitted through an $m = 1$ OAM beam produced by a helicoidally deformed parabolic antenna of the same type as used in the experiment reported in Ref. [2] and the second one with a standard Yagi-Uda antenna fed by an FM wide band signal. At the receiving end, at a distance of 520λ from the transmitting antennas, well into their far zones, the instantaneous EM fields of the $m = 0$ and $m = 1$ modes resulted in phase on one side of the central axis (azimuthal angle $\varphi = 0$)and in anti-phase at the opposite side ($\varphi = \pi$). Therefore a couple of standard linear-momentum interferometers was used to receive simultaneously and uninterruptedly discern the two OAM channels. The boxes "X" indicates the relative phase adjuster present in each of the two interferometers.

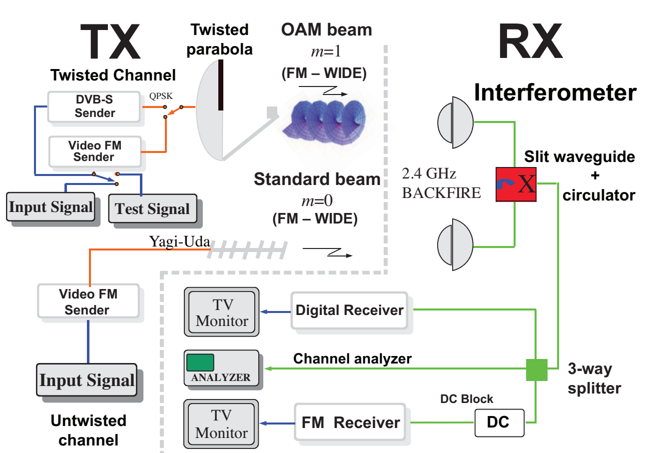

FIG. 3. Schematic description of the experimental setup for the channel capacity test. The $m = 1$ OAM beam was produced by a helicoidally deformed parabolic antenna of the same type as used in the experiment reported in Ref. [2]. This antenna was first fed by an analog FM wideband signal (27MHz) and then by a digital QPSK transmitter operating at a carrier frequency of 2.414 GHz. Horizontal linear polarization was used. The linear-momentum ($m = 0$) signal at the same carrier frequency and with the same polarization was emitted by a standard Yagi-Uda antenna, fed by an FM wide band signal. An interferometer at the receiver's end equipped with a phase adjuster device was used to discriminate and test the two channels.

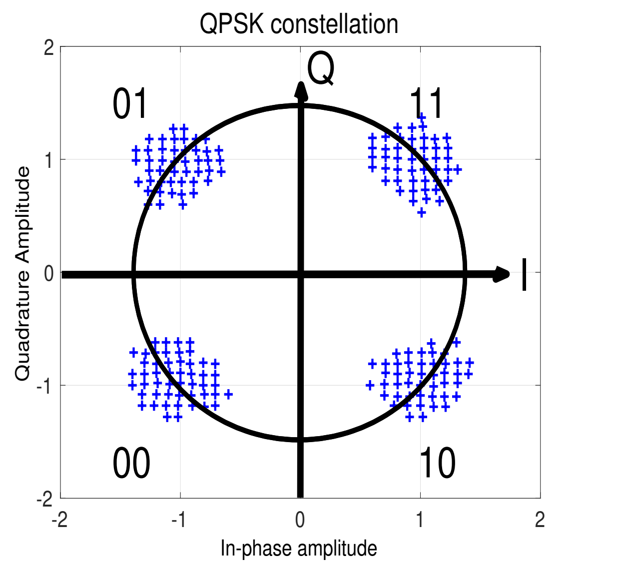

FIG. 4. Constellation diagram for the QPSK test channel carrying a DVB-S protocol in the presence of reflections and an interfering analog wide-band channel with 27 MHz frequency modulation width at the same carrier frequency.

fronts of the two beams, the field vectors of a linearly polarized $m = 1$ EM beam will be in anti-phase at two points on diametrically opposite sides of the OAM phase singularity at the centre of the two aligned beams, whereas they will be in phase for an $m = 0$ beam (see Fig. 3 and Ref. [24]). We therefore probed the phases of the fields of the received signals at such points in a standard phase interferometric manner, using two identical antennas sensitive to the linear momentum carried by the EM field. The measurements confirmed that the phases of the received signals had the expected characteristics. This allowed us to unambiguously identify and discriminate between the $m = 0$ and $m = 1$ OAM beams at the receiving end.

The digital transmitter used for the twisted ($m = 1$) mode, a Microwave Link QPSK DVB-S transmitter for the 2.4 GHz band, was tuned to 2.414 GHz (free-space wavelength $\lambda = 12.49$ cm). It transmitted live encoded video images at a rate of 11.5 Megasymbols/s. For correction purposes, we used a forward error correction of the FEC=3/4 type, meaning that after three bits transmitted, a fourth bit was added. This transmitter was connected to the twisted parabolic antenna with a four elements patch feeder

The transmitted $m = 1$ signal was encoded with a DVB-S protocol quadrature phase shift coding (QPSK)modulation. QPSK is a digital phase encoding technique used in many telecommunications applications today. It employs, at any given time, four different phase states $\{01, 11, 10, 00\}$ for the carrier. These four phase states correspond to $\{0, 90, 180, 270\}$ degrees of relative phase shifts, respectively. For each temporal period, the phase can change once, while the amplitude remains constant. In this way, two bits of information are conveyed within each time slot.

On the same UHF S-band carrier frequency, we superimposed an untwisted ($m = 0$)100 mW analogue frequency modulation (FM)transmission with the same horizontal polarization state as the OAM transmission and along the same path. The antenna used for the $m = 0$ beam was a 16–20 dBi Yagi-Uda antenna.

The QPSK constellation diagram in Fig. 4 was measured for information transfer in a 17 MHz bandwidth around the carrier frequency of 2.414 GHz in the presence of beam reflections and of the interfering untwisted 100 mW FM signal at the same carrier frequency.

The modulation error ratio (MER) of the QPSK alone was larger than 20 dB, with a bit error rate (BER) of 10^{-8} and a carrier-to-noise ratio C/N > 15 dB, including the effect of ground reflections. The average background noise power in a 100 MHz bandwidth was measured at -93 dBm, peaking at -85.6 dBm at the centre frequency. The power of the received FM signal was measured by inserting the spectrum analyzer in the reception line and was found to be -68.95 dBm in a 17 MHz wide transmission band.

At the phase interferometer, the horizontally polarized OAM-carrying electromagnetic beam was received as a superposition of the $m = 1$ direct beam and the reflected beam which, because of parity inversion, was $m = -1$ charged.

Because of the fading so produced, we the QPSK 12 dB and a resulting C/N ratio at 12 dB. The MER was ~ 12 dB and this guaranteed an acceptable signal reception. This effect corresponds to the interplay between the phase concatenation of the vortex and the separation in the channel so far obtained of a factor 2, corresponding to the 3dB variation observed in the digitally modulated channel.

When the FM analogue transmitter signal with the same carrier frequency was switched on, the ensuing interference on the digital, twisted signal was quite small and caused the MER to vary between 9 and 11 dB, the BER from 10^{-3} to 10^{-5}, and the C/N from 9 to 12 dBindicating that OAM states improve with their stable intrinsic phase properties future multiport schemes.

CONCLUSIONS

Radio transmissions with OAM states are a robust way to implement multiplexing schemes for telecommunications. Twisted waves are natural eigenmodes of the electromagnetic field and preserve their phase spatial profile when propagating in free space. One striking example is given by the detection of twisted waves from M87* black hole that revealed the rotation of this compact object located at a distance of 56 million light years far away from Earth [46].

The intrinsic robustness of OAM wavefronts with respect to external interferences can surely find applications for secure communication also under electromagnetic jamming conditions.

Periodical tests measured the separation between the two channels gave averaged value of ~ 33 dB for the first setup, ~ 25 dB for the second and ~ 28 dB for the third one. The channel capacity was characterized outdoor by inserting a 4-QAM digital transmission in the twisted channel. There we obtained a maximum channel separation value of ~ 50 dB when the conditions of phase orthogonality were fulfilled and no reflections were present.

It should be emphasized that for EM beams of the kind used in the experiment described here, OAM is distinctively different from—and independent of—SAM (wave polarization). If such a beam is already N-fold OAM encoded, adding SAM will double the information transfer capacity by virtue of the fact that the dimensionality of the state space doubles from N to $2N$. So far we have not utilized the polarization (SAM)degree of freedom in any of our OAM radio experiments. Since OAM is associated with the the phase front structure of EM waves, it is essential to investigate what impact this physical fact might have on the possibilities to phase modulate OAM radio waves and the effects of signal reflections from the environment.

Our experimental results are in full agreement with numerical simulations performed. The maximum separation in the wide-band channels was 30 dB whereas for vertical polarization we estimate it to be more than 50 dB when the receivers span the maximum of phase variation of the vortex. This clearly shows that OAM can be used to increase the transmission capacity of our common-use devices, allowing multiple services and users to share the same frequency band. We consider our experimental verification of the feasibility of using OAM radio in communications applications using phase modulation a significant leap forward, and a pivotal step toward the implementation of novel radio concepts, applications and protocols also in presence of reflections from the environment.

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